Optimal Trajectory Tracking Control for Wind Turbines During Operating Region Transitions

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Abstract—Control systems for wind turbines have become an active area of research over the past decade as the wind industry has grown and more turbines are installed. Properly sited turbines experience many transitions between below rated operation, where the goal is to extract maximum energy from the wind, and rated speed operation, where the goal is to regulate the power capture to the rated power of the turbine. Many of the largest structural loads are induced during the transition between these operating regions.

This paper focuses on using preview wind speed measurements to schedule, optimize, and track a desired trajectory of the wind turbine states and inputs during region transitions between below-rated and above-rated operation using a receding-horizon trajectory tracking control system. The goal of this control system is to reduce the structural loading on the turbine components through smoother region transitions and multivariable control. The wind speed preview measurements are used to generate an initial desired trajectory of the turbine. This desired trajectory is optimized by projecting the desired trajectory onto the turbine trajectory manifold in a weighted $L_2$ sense. The optimized trajectory is then used as a reference for a time-varying linear quadratic optimal controller.

I. INTRODUCTION

While wind energy has been a rapidly growing percentage of the global energy portfolio over the last decade, there is continuing motivation to make wind energy more cost effective through increased power capture efficiency and reductions in structural loading that can lead to longer wind turbine lifetimes. Wind turbine sizes have grown to increase the amount of wind that passes through the swept area of the rotor and to take advantage of higher wind speeds at higher distances from the ground. To minimize material costs and weight, wind turbines are now more flexible and susceptible to damaging loads from extreme winds. Protecting the components of these large, flexible wind turbines provides interesting research opportunities for control systems engineers.

The primary goals of traditional wind turbine control systems are to maximize energy production and reduce damaging loads on turbine components. Operation of wind turbine control systems is often divided into several regions. When the turbine is producing power below the rated power of the generator and components, the primary control goal is to maximize energy production, often referred to as Region 2 operation. Once the turbine has reached its rated speed $\Omega_{\text{rated}}$ and is producing rated power, the control goal is to regulate the turbine speed and power at the rated levels, often referred to as Region 3 operation. A properly sited wind turbine experiences many transitions between Regions 2 and 3. Due to speed limitations to meet noise and structural vibration constraints on large-scale wind turbines, the operating trajectories of Region 2 and Region 3 do not intersect at rated speed, so a transition region often referred to as Region 2.5 (R2.5) is implemented.

The traditional Region 2.5 transition is known to be problematic for several reasons [1]. The generator torque is actuated most rapidly due to changes in rotor speed during these transitions, inducing stress on the drivetrain. Control authority is transferred between the generator torque and blade pitch control loops during transitions between Region 2 and Region 3, as one loop is saturated while the other has control authority. Further complicating these transitions are the significant non-linearities of the blade aerodynamic sensitivity at pitch angles near the transition region, typically handled by gain scheduling.

The research presented in this paper focuses on the utilization of lidar wind preview measurements for a trajectory tracking wind turbine control system to reduce structural loads through coordinated torque and pitch control and smooth Region 2.5 transitions. An overview schematic of the control system can be seen in Fig. 1. An initial desired trajectory is scheduled from filtered wind preview measurements. The desired trajectory is then optimized to generate a regulation trajectory by projecting the desired trajectory on the turbine dynamic manifold in a weighted $L_2$ sense, effectively transforming a dynamically constrained optimization problem into an unconstrained problem. A model of the turbine is then linearized about the regulation trajectory and a linear quadratic (LQ) optimal controller is used to track the trajectory.

This paper is organized as follows. Section II introduces the basic operation of a wind turbine and the goals of traditional wind turbine control systems. Section III explains the method of scheduling a desired trajectory for the turbine when preview wind speed measurements are available. Sec-
Section IV discusses the trajectory tracking controller (TTC). Finally, Section V highlights selected simulation results, and Section VI provides concluding remarks.

II. WIND ENERGY BASICS

Wind turbines capture a fraction of the kinetic energy of the wind for conversion to electrical energy. The aerodynamic forces on the turbine blades turn the rotor which is connected to an electrical generator via a drivetrain that often has a gearbox. Most large-scale wind turbines can pitch their blades to control the power capture of the rotor and can operate at variable speeds, which reduces loads and allows for increased power capture. Power electronics must be used to connect the variable speed wind turbine to the utility grid, which allows for the generator load torque on the drivetrain to be electronically controlled in a very rapid manner [2].

The power available in a uniform wind field flowing perpendicularly through the rotor plane is

\[ P_w = \frac{1}{2} \rho A v^3 \]

where \( \rho \) is the density of air, \( A \) is the area that is swept by the blades, and \( v \) is the wind speed. The fraction of the available power that a wind turbine rotor captures is called the turbine’s power coefficient \( C_P(\beta, \lambda) \), where \( \beta \) is the collective pitch of the turbine blades and \( \lambda \) is the tip-speed ratio (TSR). The TSR is defined as the ratio of the tangential speed of the blade tips to the mean wind speed, or

\[ \lambda = \frac{\Omega_r R}{v} \]

where \( \Omega_r \) is the rotor rotational speed and \( R \) is the radius of the rotor swept area [2]. A steady-state characterization of \( C_P \) for the 5-MW turbine model used in this study [3] can be seen in Fig. 2.

The aerodynamic power \( P_a \) captured by the turbine rotor and the electrical energy produced by the generator \( P_g \) are

\[ P_a = \frac{1}{2} \rho A v^3 C_P(\beta, \lambda) \]

\[ P_g = \tau_g \Omega_r N_{gear} \eta_{eff} \]

where \( N_{gear} \) is the gear ratio of the drivetrain gearbox and \( \eta_{eff} \) is the conversion efficiency of the turbine drivetrain and generator. The simplified rotor dynamics can therefore be expressed as a nonlinear differential equation

\[ J_t \dot{\Omega}_r = \tau_a - N_{gear} \tau_g \]

\[ \tau_a = \frac{P_a}{\Omega_r} = \frac{1}{2} \rho A C_P(\beta, \lambda) v^3 \]

where \( J_t \) is the total rotational inertia of the turbine and \( \tau_a \) is the aerodynamic torque delivered to the rotor.

The commanded collective blade pitch angle \( \beta \) and generator torque \( \tau_g \) are the control inputs. In Region 2, or below-rated operation, the primary control goal is to maximize power capture by holding blade pitch constant at \( \beta \) and maintaining the TSR of the turbine at \( \lambda_s \), where \( \beta_s \) and \( \lambda_s \) are the blade pitch and TSR that produce the maximum power capture by holding blade pitch constant at \( \beta_s \) and maintaining the TSR of the turbine at \( \lambda_s \). In steady state, as shown in Fig. 2. Traditionally, the TSR \( \lambda_s \) is tracked in Region 2 by controlling the generator torque to balance the steady-state aerodynamic torque according to [2]

\[ \tau_g = k_r \Omega_r^2, \quad \text{where} \quad k_r = \frac{1}{2} N_{gear} \rho A R^3 C_P(\beta_s, \lambda_s) \lambda_s^3 \]

In traditional Region 3 control, the blades are pitched to regulate the rotor speed at rated speed and the generator torque is either held constant at rated torque or controlled to maintain rated power. The industry standard blade pitch control is a gain-scheduled proportional-integral (PI) controller that regulates the rotor speed at rated speed.

In most large-scale wind turbines, the Region 3 rated operating point does not lie on the traditional Region 2 “maximum energy capture” trajectory due to a rotor speed limitation that exists because of noise emission constraints.
Region 2 and Region 3

The wind turbine rotor has a large inertia that responds as a low-pass filter to the wind, so the rotor speed \( \Omega_g \) is scheduled by further low-pass filtering the wind speed measurements with a continuous-time first-order low-pass filter with a cutoff frequency of 0.2 Hz. These filtered wind speed measurements are then used to schedule the desired rotor speed to track the maximum power \( \lambda_\ast \) up to the rated turbine speed \( \Omega_{\text{rated}} \). The generator torque \( \tau_g \) is scheduled to follow the speed-torque trajectory seen in Fig. 3. The purpose of pitching the blades is to shed extra aerodynamic power from the rotor so that the turbine does not exceed rated speed or rated power. The blade pitch commands are scheduled by filtering the wind measurements with a zero-delay linear-phase, or boxcar, filter and using a lookup table of the steady-state blade pitch angles that are used at each wind speed by the BL-CP control system. Once the scheduling of the desired trajectory is complete, the initial conditions of the desired trajectory are set to match that of the turbine feedback, and the desired trajectory is then rate-limited and smoothed with a zero-delay discrete boxcar filter of width 0.2 sec so that the trajectory is continuous.

IV. Trajectory Tracking Control

The trajectory tracking controller (TTC) used in this study will project the desired trajectory \( \xi_d \) onto the dynamic manifold \( \mathcal{T} \) in a weighted \( L_2 \) sense to generate a regulation trajectory \( \xi_r \). A linear quadratic regulator (LQR) control system is then implemented by using time-varying feedback gains \( K_r \) to track \( \xi_r \). The upstream distance at which the lidar measures the wind speed is assumed to correspond with a finite preview horizon \( T_p \) that is dependent on the average wind speed. The scheduling and optimization of \( \xi_d \) to generate \( \xi_r \) is performed over the entire preview horizon \( T_p \). This algorithm is executed every \( T_u \) seconds so only \( K_r(0:T_u) \) need to be calculated for real-time control, where \( T_u \) is the update time and \( T_u < T_p \).

The internal model used for the TTC is a nonlinear state-space implementation of (3), with one state \( x = \Omega_r \), two control inputs \( u = [\beta, \tau_g]^T \), and a wind speed disturbance input \( v \):

\[
\dot{x} = f(x, u, v). \tag{6}
\]

The wind turbine \( C_P \) characterization, as seen in Fig. 2, is fit with a polynomial of the form

\[
C_P(\beta, \lambda) = p_{00} + p_{10} \beta + p_{01} \lambda + p_{11} \lambda \beta + p_{20} \lambda^2 + p_{21} \lambda \beta^2 + p_{22} \beta^2 \lambda + p_{12} \lambda \beta^2 + p_{02} \lambda^2 + p_{03} \lambda^3 + p_{13} \beta \lambda^3 + p_{04} \lambda^4, \tag{7}
\]

which is differentiable as long as the turbine is not at rest. The fitting of the polynomial is weighted more heavily for regions of the \( C_P \) surface that are near the desired trajectories. The 2nd by 4th order polynomial fit has been used in prior research [8] and the behavior of the nonlinear model with the polynomial fit \( C_P \) closely matched that of the
aero-elastic model used for simulation, which is described in Section V. The internal model of the control system uses the low-pass filtered wind speed measurements as the wind speed disturbance \( v \), which allows the wind turbine model to be linearized about a trajectory \( \xi = (x(\cdot), u(\cdot), v(\cdot)) \) as

\[
\dot{x} = A(x(t), u(t), v(t)) x + B(x(t), u(t), v(t)) u
\]

where each coefficient in (9) is time-varying and the aerodynamic torque \( \tau_a \) is dependent on the measured wind speed disturbance \( v \). The turbine model is smooth on \( C^3 \) for the region of interest, allowing for calculation of the second derivative \( D^2 f(x, u, v) \) for use in the projection operator calculations.

The goal of projecting the desired trajectory \( \xi_d \) onto the dynamic manifold \( T \) is to generate a regulation trajectory \( \xi_r \) that can be used in an unconstrained optimal LQ controller. The projection is performed in a weighted \( L_2 \) sense by finding a trajectory to locally minimize the least squares cost functional

\[
h(\xi) = \frac{1}{2} \int_{\tau_0}^{\tau_f} \left( ||x(\tau) - x_d(\tau)||^2_{Q_e} + ||u(\tau) - u_d(\tau)||^2_{R_e} \right) d\tau
\]

subject to \( \dot{x} = f(x, u, v) \) given symmetric positive definite matrices \( Q_e \) and \( R_e \).

The projection operator based descent method proposed in [9] is used to generate \( \xi_r \in T \), where Newton’s method is used to iteratively update the desired trajectory. This method entails linearizing about the desired trajectory and calculating the LQ feedback gains \( K_r \) by solving a linear quadratic optimal control problem [10]

\[
K_r(t) = R_r^{-1} B(t)^T P_r(t)
\]

where \( P_r(\cdot) \) satisfies the Riccati equation [10]

\[
P_r + A(t)^T P_r + P_r A(t) - K_r(t)^T R_r K_r(t) + Q_r = 0
\]

\[
P_r(T_p) = P_{tr}
\]

with \( Q_r = Q_r^T > 0, R_r = R_r^T > 0, P_{tr} = P_{tr}^T > 0 \). It should be noted the subscript \( r \) indicates the use in the regulator equations, and that \( Q_r \) and \( R_r \) do not need to be the same as the \( Q_e \) and \( R_e \) in the cost function \( h(\cdot) \) in (10). The LQ feedback gains are used to regulate the internal controller model about \( \xi_d = f(x_d(\cdot), u_d(\cdot)) \), which generates a trajectory \( \xi_n \in T \) where \( \xi_n = P(\xi_d) \) and \( P \) is the nonlinear projection operator to the turbine trajectory manifold [9]. This process can be expressed as the integration of the nonlinear feedback system

\[
\begin{align*}
x_n &= f(x_n, u_n, v), \quad x_n(0) = x_0 \\
u_n &= u_d(t) + K_r(t)[x_d(t) - x_n]
\end{align*}
\]

The desired trajectory is then updated to \( \xi_{d+} \) by taking a step along the gradient of \( h(\xi_n) \) using Newton’s method, as described in [9]. The process is then repeated with \( \xi_{d+} \). The model is linearized about \( \xi_{d+} \), the LQ feedback gains for \( \xi_{d+} \) are calculated and used to generate \( \xi_{n+} \), and the gradient of \( h(\xi_{n+}) \) is again evaluated to update the new desired trajectory \( \xi_{d+} \). This process is repeated until either the gradient of (10) is sufficiently small or 20 iterations have elapsed, at which point the updated trajectory is considered to be the regulation trajectory \( \xi_r \). The internal model is linearized about \( \xi_r \) to calculate the LQ feedback gains \( K_r \). Then \( \xi_r(0 \cdot T_u) \) and \( K_r(0 \cdot T_u) \) are passed through a buffer to the real-time system to track the desired trajectory for the duration of the algorithm update time \( T_u \).

It is traditionally desirable to keep the blades pitched at \( \beta_s \) when operating in Region 2 and keep the rotor speed and the generator torque near the rated operating point when operating in Region 3, as discussed in Section II. The values of \( Q_c, Q_r, R_c, \) and \( R_r \) are therefore scheduled based on the mean wind speed over the algorithm update time \( T_u \) of the optimization algorithm to ensure that these desired constraints are appropriately weighted in the generation of \( \xi_r \) and the LQ feedback gain \( K_r \). These matrices were chosen to be diagonal and the scheduling of the diagonal entries was performed by linearly interpolating between their Region 2 \((\omega(0,T_u) \leq 10.5 \text{ m/s})\) and Region 3 \( (\omega(0,T_u) \geq 12.5 \text{ m/s}) \) values. The penalties in \( Q_c \) and \( Q_r \) on rotor speed deviation are increased more significantly as the mean wind speed increases into Region 3 to put a strong emphasis on regulating rotor speed at rated to avoid excessive rotor over-speed, which may cause an emergency shutdown of the turbine that induces extremely large loads on the turbine components.

The receding horizon implementation of the TTC with wind speed dependent \( Q \) and \( R \) penalties may introduce step changes into the regulation trajectory \( \xi_r \) and the feedback gains \( K_r \) that are used to control the turbine every \( T_u \) seconds. The generator torque \( \tau_g \) and blade pitch \( \beta \) commands are smoothed with a 0.2 second boxcar filter. The generator torque command is passed through an additional low-pass filter with a cutoff frequency below the natural frequency of the drivetrain and a notch at the side-side tower moment because the generator load torque is directly applied to the drivetrain which can rapidly change the rotor rotational speed and potentially induce a side-side moment on the tower.

V. SIMULATION RESULTS

This section describes the simulation environment that was used to evaluate the developed control system and highlights selected simulation results. Simulations were performed using a 5-MW aero-elastic turbine modeled in FAST code, which was developed by the National Renewable Energy Laboratory (NREL). More information regarding FAST can be found in [11], and a detailed description of the 5-MW turbine model including the baseline collective-pitch (BL-CP) controller can be found in [3]. The simulations are run with the blade flap and edgewise, tower fore-aft and side-side, drivetrain torsion and generator rotation degrees of freedom (DOFs) enabled. Enabling these DOFs allows for a more accurate analysis of the damage equivalent loads (DELs) [12] that are induced on the turbine components during simulation. The 5-MW model is also augmented with a second-
order low-pass blade pitch actuator model. The simulations are performed with a time step of $T_a = 0.0125$ seconds, so that the simulation sampling frequency is more than an order of magnitude greater than the natural modes of the turbine components. The trajectory tracking controller (TTC) described in Section IV is compared against the BL-CP and BL-SOA controllers, as well as a BL-CP controller that is augmented with feedforward pitch commands, referred to as the feed-forward collective pitch controller (FF-CP). The FF-CP control system filters the wind speed measurements and uses a steady-state operating point lookup table to prescribe a blade pitch command which is added to the feedback PI pitch command, as described in [13]. The implementation of the TTC uses a preview horizon of $T_p = 10$ seconds and an algorithm update time of $T_u = 4$ seconds, which are realistic values for the turbine model.

The controllers are simulated with two different types of wind fields. The first is an International Electrotechnical Commission (IEC) extreme wind gust at 10 m/s, shown in Fig. 4. The IEC is an organization that sets wind turbine standards, and this gust has been classified as an extreme event that induces very large structural loads and is often used for performance evaluations of wind turbine controllers. The controller receives perfect wind speed measurements when simulating with the extreme gust. The control system trajectories can also be seen in Fig. 4. The TTC produced the trajectory with the smallest rotor overspeed and the smoothest torque trajectory by pitching the blades rapidly due to the large gust. The TTC and FF-CP control systems actuate the blade pitch upon measuring the incoming wind gust, responding before the rotor speed exceeds the rated speed, unlike the two baseline control systems which actuate the blade pitch after the turbine reaches the rated speed.

Simulations were also performed with 5 turbulent, full field wind files that are generated with the TurbSim code and have a mean wind speed of 10.7 m/s and uses the Great Plains Low-Level Jet spectral model, a vertical power law shear model with a shear exponent of 0.134, a Gradient Richardson number of 0.02, a friction or shear velocity of 0.5 m/s, and a mixing layer depth of 808.072 m as defined in [14]. Realistic lidar preview wind speed measurements are also used instead of perfect preview measurements, with the simulated lidar located in the center of the rotor hub taking line-of-sight measurements along 3 beams that sample the wind with a spatial range-weighting filter [6]. The lidar beams are focused at 75% blade span (a radius of 47.25 m) at a distance of 107 m upwind of the turbine. The lidar configuration was chosen based on the research presented in [6] and provides a preview of $T_p = 10$ seconds based on the mean speed in the wind fields. The lidar beams scan a circular pattern in front of the turbine, measuring the wind at the projected locations that the blades will be after $T_p$ seconds if the rotor speed is constant over the preview horizon.

The measurements from the three beams are averaged and low-pass filtered to account for high-frequency measurement error and also to remove the periodic fluctuations in the averaged measurement when spinning in an exponential wind shear model. Fig. 5 shows the performance metrics of the TTC compared to the two baseline feedback controllers and the baseline feedforward controller when simulating with turbulent wind fields. It can be seen in Fig. 5 that the TTC succeeds at improving all of the performance metrics except for the low speed shaft DEL when compared to the BL-CP controller. The TTC does not reduce the tower side-side DELs more than the BL-SOA or FF-CP controllers, but does provide the largest reductions in blade root moment and tower fore-aft DELs.

**VI. DISCUSSION AND CONCLUSIONS**

This paper explains a method for implementing a trajectory tracking controller (TTC) which uses preview measurements in a projection operator based trajectory optimization and a receding horizon time-varying LQ control system. The desired trajectory $ξ_d$ is scheduled over the preview horizon using filtered wind preview measurements. The desired trajectory $ξ_d$ is then projected onto the trajectory manifold of the turbine in a weighted $L_2$ sense, which is then used as the regulation trajectory $ξ_r$. The turbine model is linearized about $ξ_r$ to calculate the time-varying LQ optimal feedback gains and $ξ_r$ is used as the reference for the trajectory tracking controller.

Many parameters can be adjusted and optimized in this overall control algorithm. The transition trajectory and low-pass filters used for generating an initial desired trajectory $ξ_d$ were chosen by analyzing the ramp responses of an offline dynamic programming algorithm, but the scheduling method...
DELs and performance measures for the FAST 5-MW turbine model with various controllers averaged across simulations with 5 turbulent wind fields of 600 seconds each. The performance metrics are normalized to the baseline collective pitch ‘BL-CP’ controller described in [3]. Values less than 100% are improvements in all measures. The industry state-of-the-art ‘BL-SOA’ feedback controller is described in [5]. The ‘FF-CP’ control system is the ‘CP-BL’ control system with feedforward pitch commands as described in [13]. The ‘TTC’ controller is the control system described in this paper. The percent over-speeds normalized to rated speed are 2.3%, 3.9%, 3.7%, and 1.6% for the BL-CP, BL-SOA, FF-CP, and TTC, respectively.

The control system presented in this paper provides smoother transitions between operating regions and reduces loads and rotor overspeed during Region 2.5 transitions. Future work includes further optimizing the trajectory generation for more DEL reductions. Augmenting the internal model so that the inputs are the rate of change of blade pitch and generator torque can allow penalties to be placed on actuator effort. The internal model can also be augmented with a second-order tower model so that a penalty can be placed on the tower fluctuations.

REFERENCES