Quantum Error Correction

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Agenda

- Introduction
- Basic concepts
- Error Correction principles
- Quantum Error Correction
- QEC using linear optics
- Fault tolerance
- Conclusion

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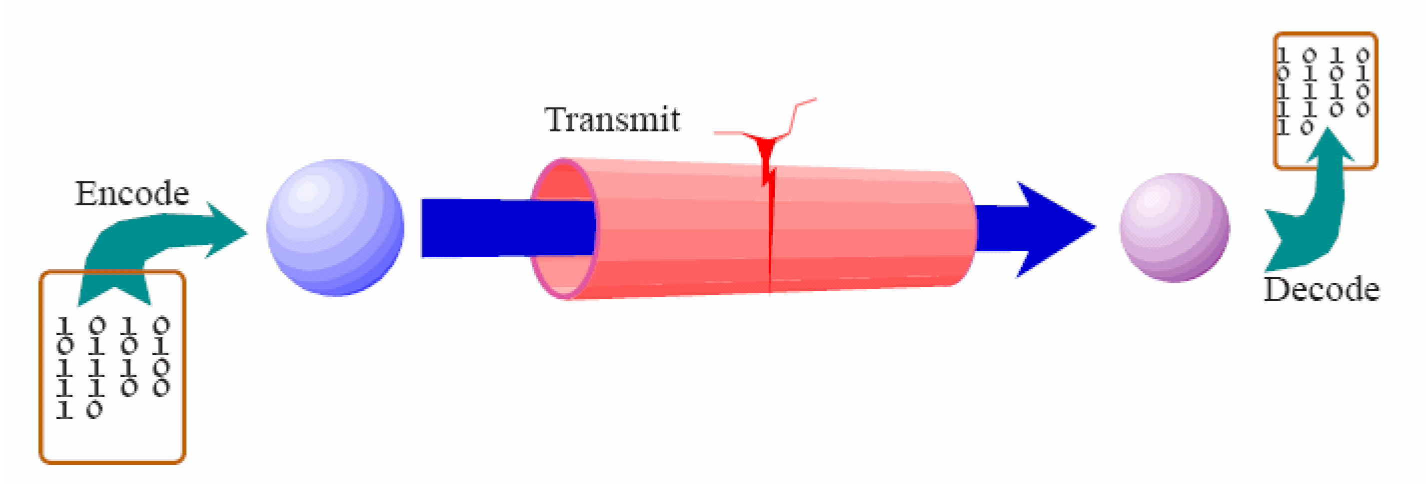
Introduction to QEC

- Basic communication system



- Information has to be transferred through a noisy/lossy channel
- Sending raw data would result in information loss
- Sender encodes (typically by adding redundancies) and receiver decodes

- QEC secures quantum information from decoherence and quantum noise

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Two bit example

Error model:

Errors affect only the first bit of a physical two bit system

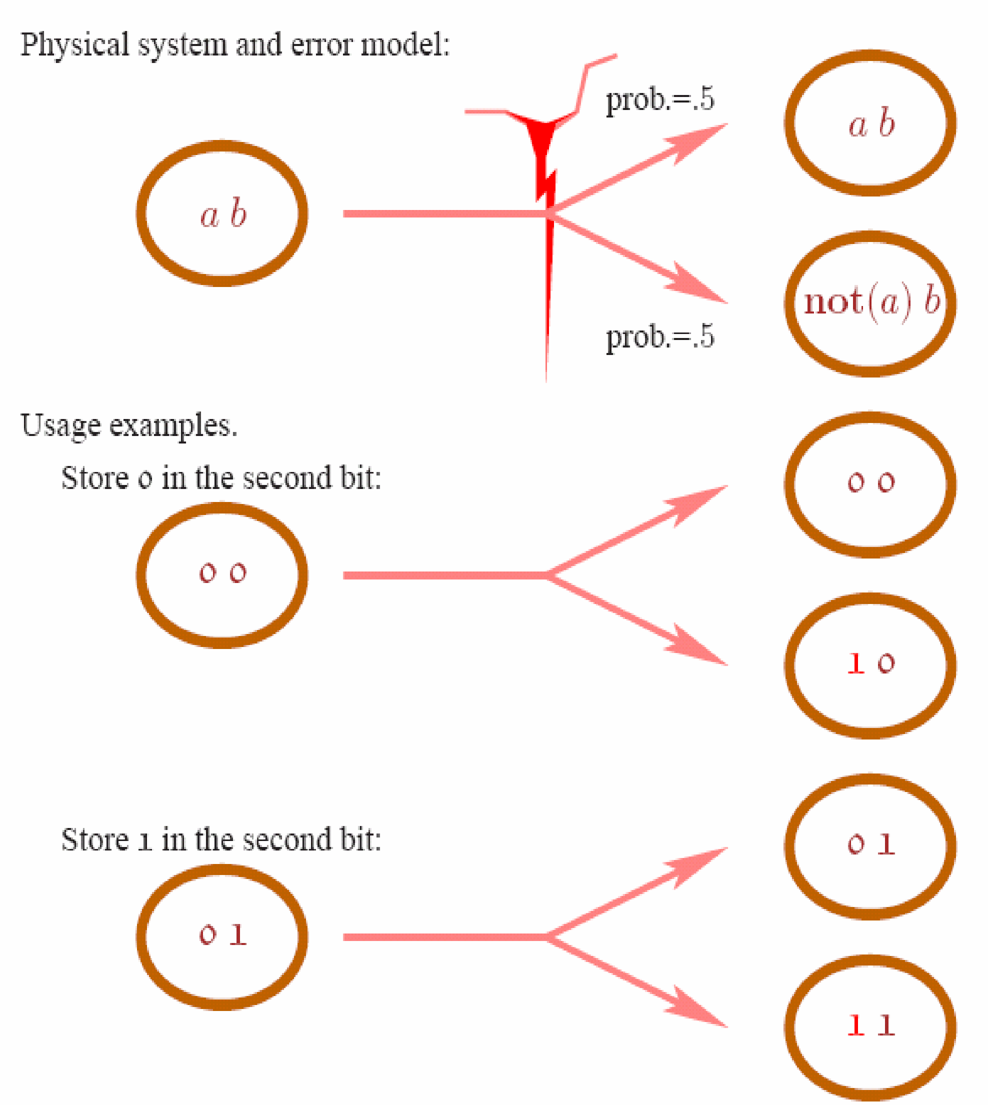
Redundancy:

States 0 and 1 are represented as 00 and 01

Decoding:

00 $\longrightarrow$ 0
10 $\longrightarrow$ 0
01 $\longrightarrow$ 1
11 $\longrightarrow$ 1

Subsystems: Syndrome, Info.



Repetition Code

Representation:

$0 \longrightarrow 000$

$1 \longrightarrow 111$

Majority decoding

Error Model:

Independent flip probability = 0.25

Analysis:

- 1 bit flip – *No problem!*
- 2 (or) 3 bit flips – *Ouch!*

Error Probabilities:

2 bit flips: 0.25 *0.25 * 0.75

3 bit flips: 0.25 * 0.25 * 0.25

Total error probabilities:

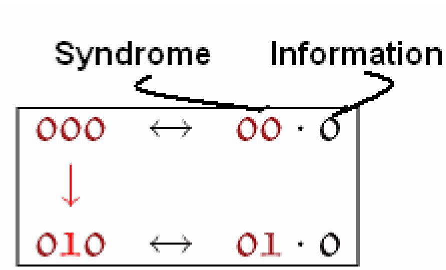
With repetition code:

0.25^3 + 3 * 0.25^2 * 0.75 = **0.15625**

Without repetition code:

0.25

Improvement!



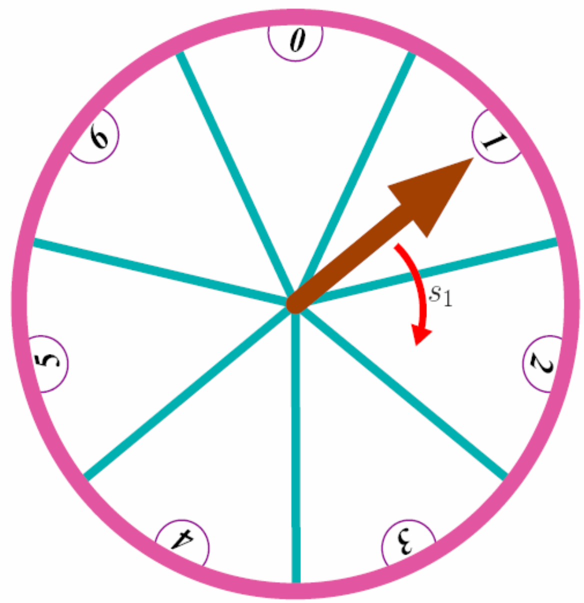
Cyclic system

States: 0, 1, 2, 3, 4, 5, 6

Operators:

$$s_1(l) = l + 1 \text{ for } 0 \le l \le 5$$

$$s_k = \underbrace{s_1 \dots s_1}_{k \text{ times}}, \qquad s_{-k} = s_k^{-1}$$



map $0 \to 1, 1 \to 4$.

Error model:

s_k probability = qe^{-k^2} where q = 0.5641

$$\sum_{k=-\infty}^{\infty} qe^{-k^2} = 1$$

s_0 probability = 0.5641

s_{-1} and s_1 probability = 0.2075

Correct detection probability = 0.9792

Decoding

0	$\to$	0
1	$\to$	0
2	$\to$	0
3	$\to$	1
4	$\to$	1
5	$\to$	1
6	$\to$	fail

Subsystem

0	$\leftrightarrow$	$-1 \cdot 0$
1	$\leftrightarrow$	$0 \cdot 0$
2	$\leftrightarrow$	$1 \cdot 0$
3	$\leftrightarrow$	$-1 \cdot 1$
4	$\leftrightarrow$	$0 \cdot 1$
5	$\leftrightarrow$	$1 \cdot 1$

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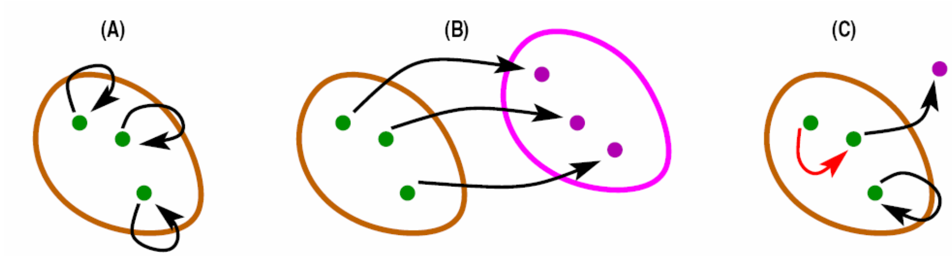
Error Correction principles

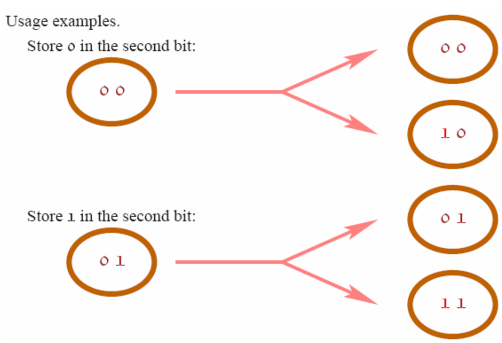
- Establish properties of the physical system
  - State space structure
  - Means of control
  - Type of information to be processed
  - Error model
- Encode information with codes in the subspace of the physical system
- Determine decoding procedure
  - Assume that the information has been modified
  - Identify “Syndrome” and “Information” subsystems
- Analyze error behavior of the code (used in encoding) and subsystem

Error detection

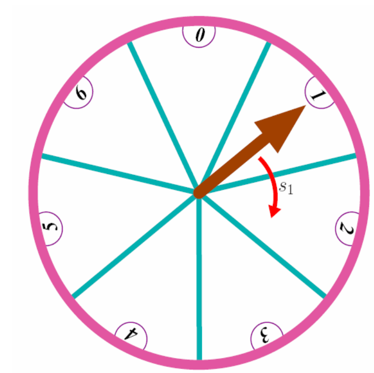
- Encoded information is transmitted
- Receiver checks whether the state is still in the code



- Detectable and undetectable errors



$$1 \longrightarrow 111$$
$$0 \longrightarrow 000$$



Theorem. E is detectable by a code if and only if for all $x \neq y$ in the code, $Ex \neq y$.

Error detection to correction

Given a code C and a set of error operators $\mathcal{E} = \{\mathbb{1} = E_0, E_1, E_2, \ldots\}$

Theorem. $\mathcal{E}$ is correctable by C if and only if for all $x \neq y$ in the code and all i, j, it is true that $E_i x \neq E_j y$.

- Necessity proof

  suppose that for some $x \neq y$ in the code and some i and j, we have $z = E_i x = E_j y$.
  If the state z is obtained after an unkown error in $\mathcal{E}$, then it is not possible to determine whether the original code word was x or y, because we cannot tell whether E_i or E_j occurred.

- Sufficiency proof

  we assume it and construct a decoding method $z \to \text{dec}(z)$. Suppose that after an unknown error occurred, the state z is obtained. There can be one and only one x code for which some $E_{i(z)} \in \mathcal{E}$ satisfies the condition that $E_{i(z)} x = z$. Thus x must be the original code word and we can decode z by defining $x = \text{dec}(z)$.

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Two Qubit example

Error model: Randomly apply Identity or Pauli operators to the first qubit

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$|\psi\rangle_{12} \rightarrow \begin{cases} \mathbb{1}|\psi\rangle_{12} & \text{Prob. } .25 \\ \sigma_x^{(1)}|\psi\rangle_{12} & \text{Prob. } .25 \\ \sigma_y^{(1)}|\psi\rangle_{12} & \text{Prob. } .25 \\ \sigma_z^{(1)}|\psi\rangle_{12} & \text{Prob. } .25 \end{cases},$$

Encoding: Realize an ideal qubit as a 2D subspace of physical qubits

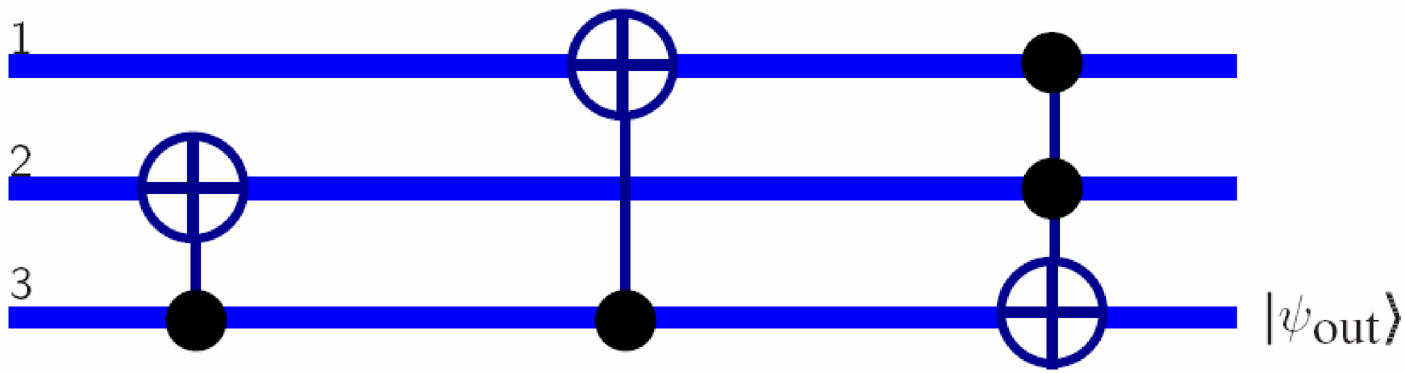
$$|\psi\rangle \rightarrow |0\rangle_1 |\psi\rangle_2$$

Decoding: Discard quibit 1 and retain qubit 2

Quantum repetition code

Error model: Independent flip error probability = 0.25 $\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$

Decoding: Majority logic. *Careful!* Need to preserve quantum coherence!!



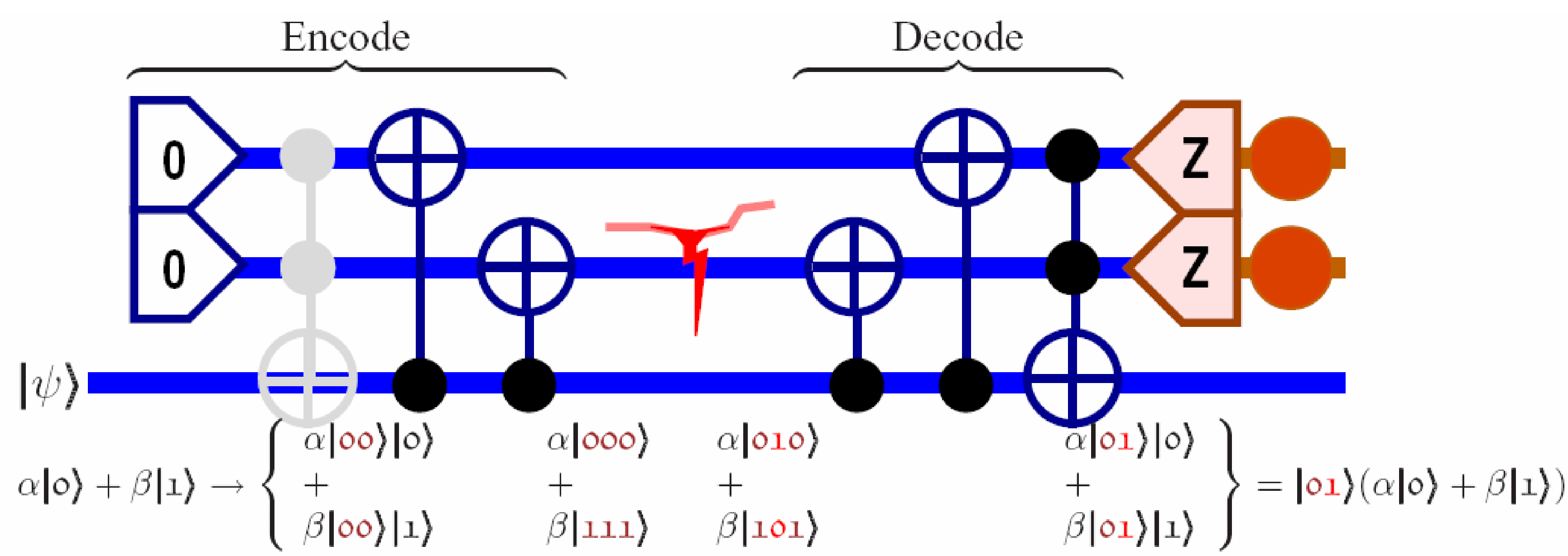
$\lvert 000\rangle$ $\lvert 001\rangle$ $\lvert 010\rangle$ $\lvert 100\rangle$	$\lvert 000\rangle$ $\lvert 011\rangle$ $\lvert 010\rangle$ $\lvert 100\rangle$	$\lvert 000\rangle$ $\lvert 111\rangle$ $\lvert 010\rangle$ $\lvert 100\rangle$	$\lvert 00\rangle\lvert 0\rangle$ $\lvert 11\rangle\lvert 0\rangle$ $\lvert 01\rangle\lvert 0\rangle$ $\lvert 10\rangle\lvert 0\rangle$
$\lvert 111\rangle$ $\lvert 110\rangle$ $\lvert 101\rangle$ $\lvert 011\rangle$	$\lvert 101\rangle$ $\lvert 110\rangle$ $\lvert 111\rangle$ $\lvert 001\rangle$	$\lvert 001\rangle$ $\lvert 110\rangle$ $\lvert 011\rangle$ $\lvert 101\rangle$	$\lvert 00\rangle\lvert 1\rangle$ $\lvert 11\rangle\lvert 1\rangle$ $\lvert 01\rangle\lvert 1\rangle$ $\lvert 10\rangle\lvert 1\rangle$

Quantum repetition code

Encoding network:

Reverse decoding network and initialize qubits 2 and 3 in the state |00>

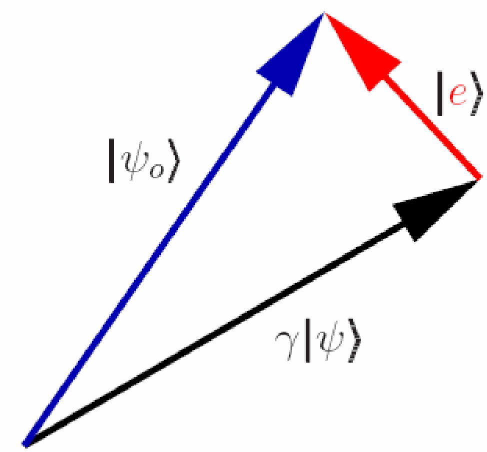
Complete quantum network:



Performance measures

- Compare output $|\psi_o\rangle$ with input $|\psi\rangle$ to determine error

$$|\psi_o\rangle = \gamma|\psi\rangle + |e\rangle$$



- Upper limit of error probability: $\epsilon = ||e\rangle|^2$
- Fidelity: $1 - \epsilon$

Example:

$$|0\rangle \xrightarrow{\text{encode}} |000\rangle \longrightarrow \begin{cases} .75^3 : & |000\rangle, \\ .25 * .75^2 : & |100\rangle, \\ .25 * .75^2 : & |010\rangle, \\ .25 * .75^2 : & |001\rangle, \\ .25^2 * .75 : & |110\rangle, \\ .25^2 * .75 : & |101\rangle, \\ .25^2 * .75 : & |011\rangle, \\ .25^3 : & |111\rangle \end{cases} \xrightarrow{\text{decode}} \begin{cases} .4219 : & |00\rangle \cdot |0\rangle, \\ .1406 : & |10\rangle \cdot |0\rangle, \\ .1406 : & |01\rangle \cdot |0\rangle, \\ .1406 : & |11\rangle \cdot |0\rangle, \\ .0469 : & |11\rangle \cdot |1\rangle, \\ .0469 : & |01\rangle \cdot |1\rangle, \\ .0469 : & |10\rangle \cdot |1\rangle, \\ .0156 : & |00\rangle \cdot |1\rangle \end{cases}$$

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QEC using linear optics

Paper:

Demonstration of Quantum Error Correction using Linear Optics

T.B. Pittman, B.C Jacobs, and J.D. Franson
Johns Hopkins University, Applied Physics Laboratory, Laurel, MD 20723
(Dated: February 7, 2005)

We describe a laboratory demonstration of a quantum error correction procedure that can correct intrinsic measurement errors in linear-optics quantum gates. The procedure involves a two-qubit encoding and fast feed-forward-controlled single-qubit operations. In our demonstration the qubits were represented by the polarization states of two single-photons from a parametric down-conversion source, and the real-time feed-forward control was implemented using an electro-optic device triggered by the output of single-photon detectors.

Encoding:

$$|0\rangle \rightarrow |0_L\rangle \equiv \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|1\rangle \rightarrow |1_L\rangle \equiv \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

Value of the logical bit corresponds to the parity of the two physical qubits

Quantum Circuit

- Single-photon qubit value is measured in the computational basis
- Assume a Z-measurement occurs on either of the two photons
- If (value = 0)

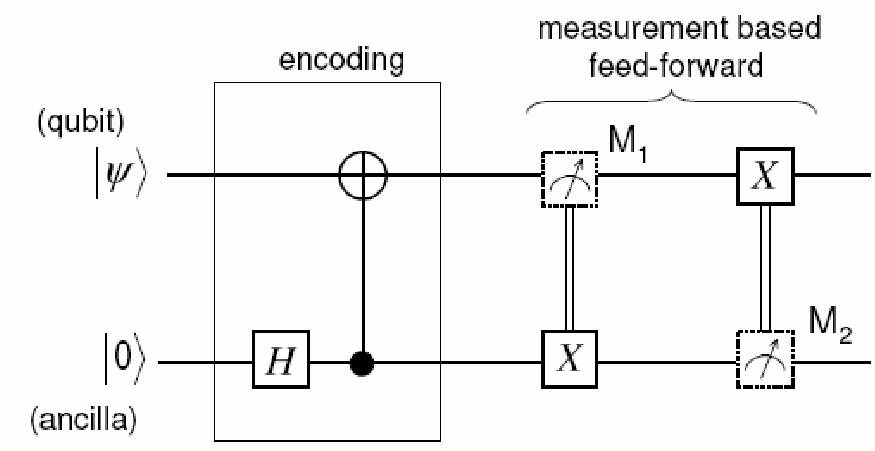
  State of the “other” photon = initial single photon qubit

  else

  State of the “other” photon = bit flipped value of the initial qubit
- In the latter case, a feed-forward-controlled bit-flip is used
- Represent qubits by the polarization states of two single photons from a parametric down conversion pair

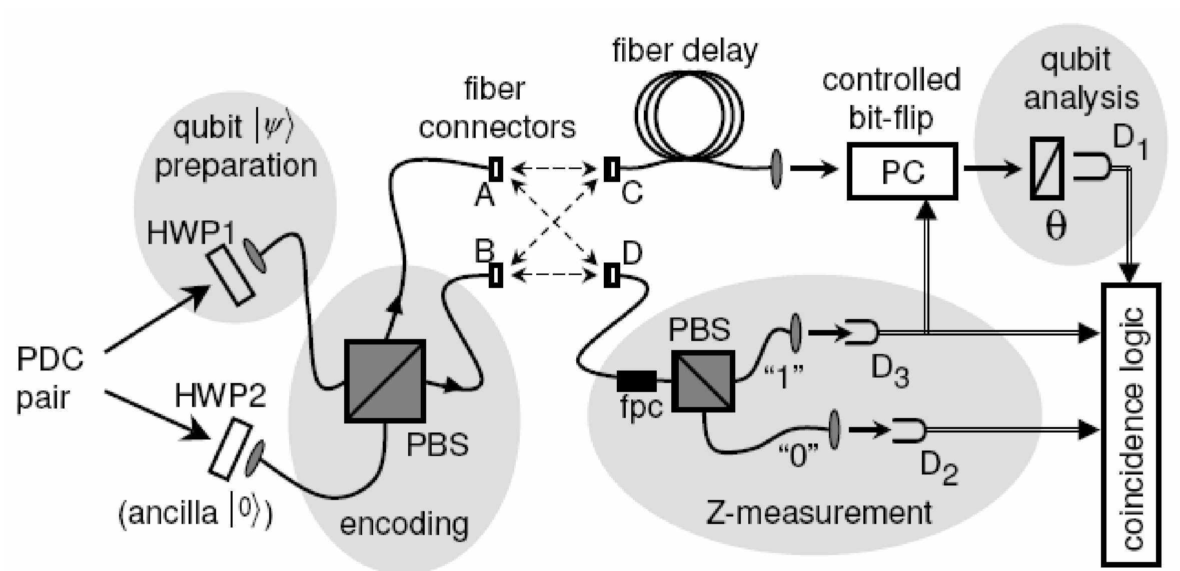
Quantum Circuit

- Encoder encodes a single-photon qubit $|\psi\rangle$ into the two photon logical qubit $|\psi_L\rangle$
- Encoding is done probabilistically using linear optics
- Feed-forward-controlled bit flip was accomplished using an electro-optic polarization rotator (Pockels cell)
- Intentionally inflict a Z-measurement on one of the photons and verify the success of QEC by comparing the corrected polarization state with the input state



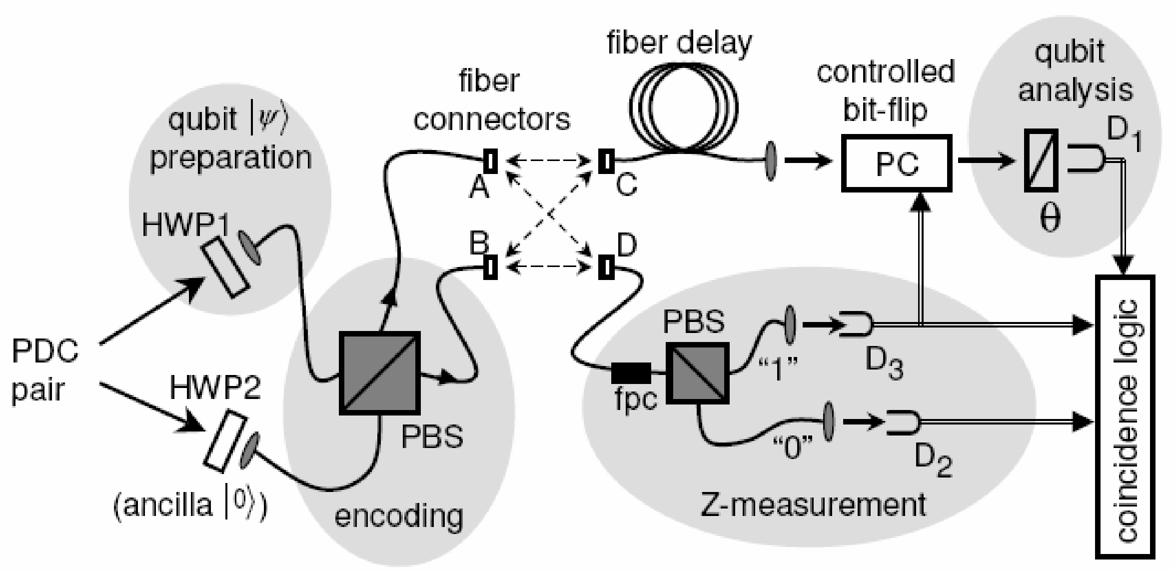
Experiment

- PDC produces horizontal SOP photons at 780nm
- HWP2 fixed at 22.5 degrees (Ancilla SOP = 45 degree linear)
- HWP1 is used for qubit $|\psi\rangle$ preparation
- Encoding can be understood as a 2-photon quantum interference effect

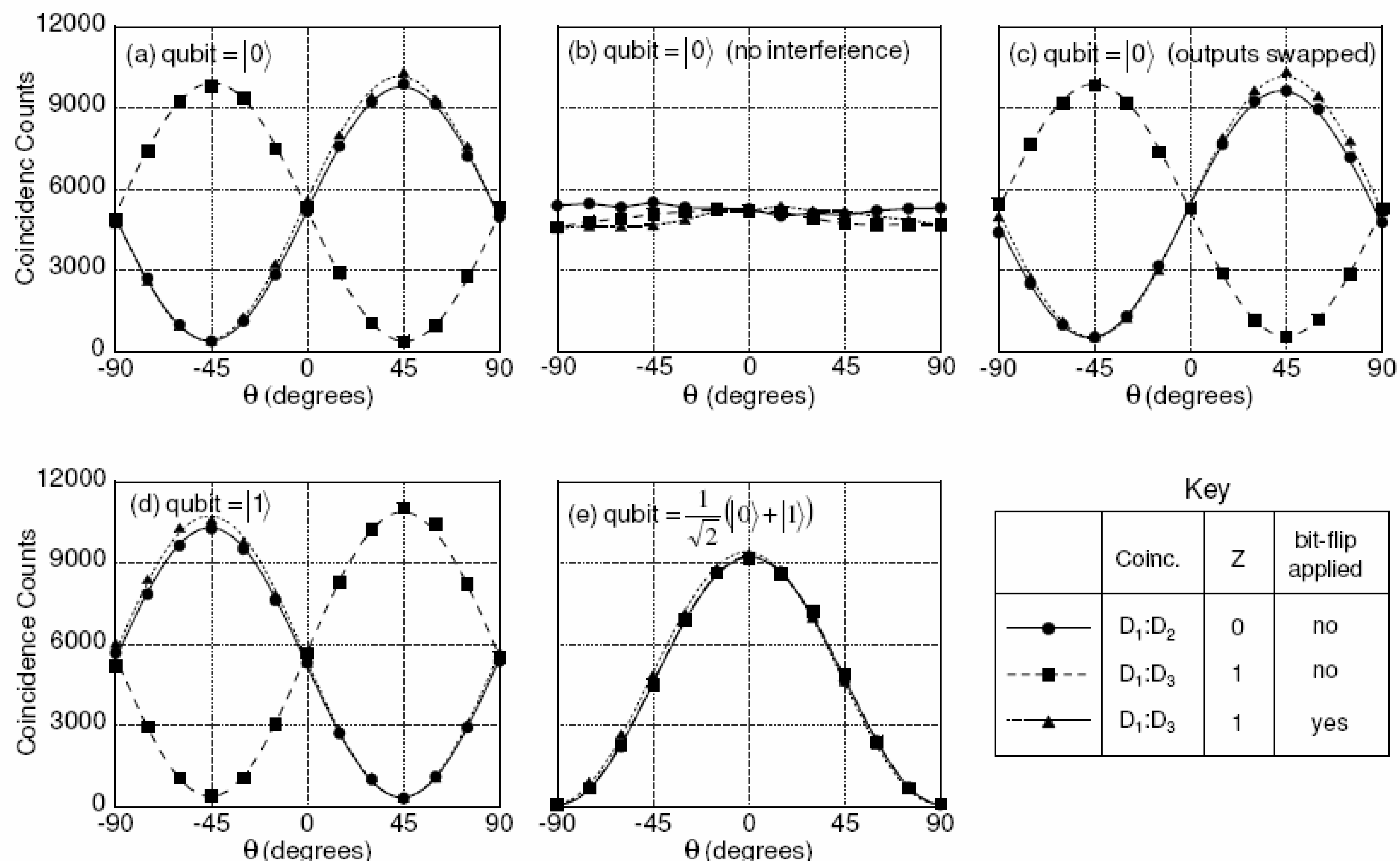


Experiment

- Fiber connector used to make a Z-measurement on either of the photons
- Fiber polarization controller makes sure that the axes of PBS corresponds to the computational basis $|\psi\rangle$
- 30m fiber delay used as feed-forward control took ~100ns
- Coincidence logic records only events in which one photon was detected by Z-measurement detectors and the second photon was detected by D1



Results



(a) qubit = |0⟩
(b) qubit = |0⟩ (no interference)
(c) qubit = |0⟩ (outputs swapped)
(d) qubit = |1⟩
(e) qubit = $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
Coincidenc Counts
Coincidence Counts
θ (degrees)
12000
9000
6000
3000
0
-90
-45
0
45
90
Key
Coinc.
Z
bit-flip applied
D1:D2 0 no
D1:D3 1 no
D1:D3 1 yes

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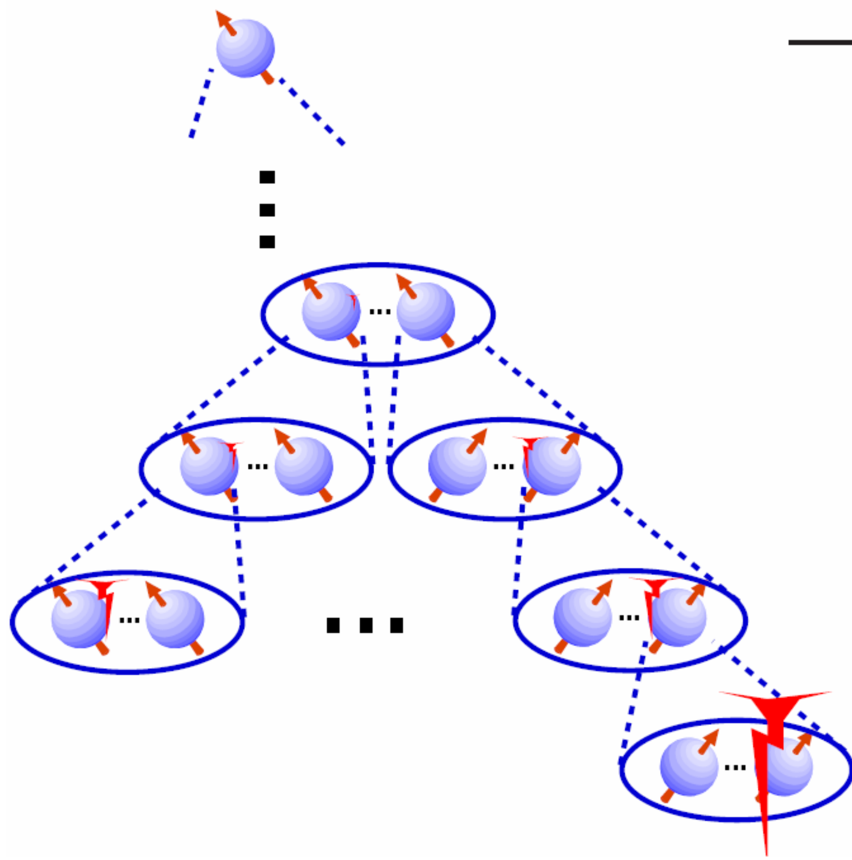
Realizing fault tolerance

- Quantum error correcting codes can be used at every successive stage for achieving low error rates



Level	Error rate
k	$\leq C^{2^k-1}p^{2^k}$
$\vdots$	
4	$p_4 \leq C(C^3p^{2^2}$ $= C^{2^3-1}p^{2^3}$
3	$p_3 \leq C(Cp^2)^2$ $= C^{2^2-1}p^{2^2}$
2	$p_2 \leq Cp^2$
1	p

Scalable QIP requirements

- Scalable physical systems
  - System must be able to support any number of independent qubits
- State preparation
  - Must be able to prepare any qubit in the standard initial state
- Measurement
  - Ability to measure any qubit in the logical basis
- Quantum control
  - Universal set of unitary gates acting on a small number of qubits
- Errors
  - Error probability per gate should be below threshold
  - Satisfy independence and locality properties

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Conclusion

- Probability of error in quantum computing/communication can be largely reduced by using error coding and correction algorithms
- Efficient linear optics implementation of QEC is possible
- Advancements in QEC and fault tolerant QIP show that "**in principle**" scalable quantum computation is achievable

References

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Acknowledgements

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http://moisl.colorado.edu

CDMOptics

http://cdm-optics.com

- Thank You!