Better Generalization in IC3

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Outline

1. Problem
2. Solution
3. Results
4. Analysis
5. Conclusions
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IC3 [Bradley 2010,2011]

- Model checking algorithm for invariance properties
- Attempts to construct an inductive strengthening of the property
- Construction is incremental: derives many simple lemmas
- Lemmas generation either:
  - Results in an inductive strengthening
  - Guides the search to a counterexample trace
- SAT-based: performs many relatively easy SAT queries
Generalization

- Key component of IC3
- Lifts IC3 from explicit to symbolic
- More successful generalization $\iff$ Fewer individual states examined

*What does IC3 generalize?*
Generalization

- Key component of IC3
- Lifts IC3 from explicit to symbolic
- More successful generalization $\Leftrightarrow$ Fewer individual states examined

*What does IC3 generalize?*
Overview of IC3

- Prove the property by induction:
  - All initial states satisfy the property
  - All successors of good states are good
Overview of IC3

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  - All initial states satisfy the property
  - All successors of good states are good
Counterexamples to Induction (CTIs): The Troublemakers

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Counterexamples to Induction (CTIs): The Troublemakers
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A state is unreachable within \( k \) steps to

A set of states is unreachable within \( k \) steps
How does generalization work?

For each state-bit:

- Drop bit
- Find the smallest superset of states that have no predecessors outside of it (if exists)
Successful Generalization
Successful Generalization

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Successful Generalization
Successful Generalization
Successful Generalization
Failed Generalization
Failed Generalization
Failed Generalization

Problem: Doubling the Hamming weight
Solution: Introduce CTI
Results: Successful generalization
Analysis: CTI eliminates self-loops
Conclusions: Better generalization in IC3
Failed Generalization

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Better Generalization in IC3
Failed Generalization

CTI
Ineffective Generalization
Counterexamples to Generalization (CTGs)
Counterexamples to Generalization (CTGs)
Counterexamples to Generalization (CTGs)
Counterexamples to Generalization (CTG)

- State preventing some generalization (dropping a specific state-bit)
- Unlike CTIs, not necessarily backward reachable
- Blocking CTGs:
  - Backward reachable: if deep, saves IC3 explicit traversal
  - Neither forward nor backward: never addressed by IC3 but could continue to obstruct generalization
ctgDown

- Instead of joining CTG with cube, turn attention to CTG
- Like CTIs, prove unreachable within $k$ steps
- If successful: generalize CTG, re-attempt CTI generalization
- If failed: join
Instead of joining CTG with cube, turn attention to CTG if limit is not exceeded
Like CTIs, prove unreachable within $k$ steps
If successful: generalize CTG, re-attempt CTI generalization
If failed: or exceeded maxCTGs limit, join, reset maxCTGs limit
Resetting Limit After Joins
Resetting Limit After Joins
Resetting Limit After Joins
Resetting Limit After Joins
Resetting Limit After Joins
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Experimental Setup

- HWMCC’10+11+12 (beemb substituted by beemf)
- 900s timeout
- Ilmc and ABC
- Light-weight preprocessing
- 5 random seeds
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# Outline

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Purpose

- Confirm reduction in length of explicit backward search
- Understand effect on various IC3 metrics
Depth of CTGs vs. CTIs

![Graph showing the relationship between average CTI depth and average CTG depth. The graph includes symbols representing worse and better performance.](image)

- Average CTI Depth vs. Average CTG Depth
- Symbols indicate performance:
  - Better Performance (×)
  - Worse Performance (+)

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Effect on Maximum Depth of Priority Queue

![Graph showing the effect on maximum depth of priority queue. The x-axis represents IImc, and the y-axis represents IImc with ctgDown. The graph compares worse performance (+) and better performance (×).]
Effect on Average Clause Size

![Graph showing the effect on average clause size with IImc with ctgDown and IImc, comparing worse and better performance.]
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Conclusions

- Useful to divert IC3’s attention to address reason for failure of generalization
- Not too aggressive handling of CTGs so as not to lose property focus
- Decreases depth of explicit search
The End

Thank you.